A Neural Network Extension of the Lee-Carter Model to Multiple Populations
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Agenda

• Introduction

• Deep Learning – Brief Overview

• Our Approach

• Discussion and Conclusion
Disclaimer

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Introduction

• Mortality rates and mortality improvement rates = key inputs into life insurance models

• Former usually based on experience of similar portfolios; latter often based on forecasting population mortality rates

• Foundational model for mortality forecasting is the Lee-Carter model (Lee and Carter 1992) (LC model)

• Many other approaches; within actuarial literature see Cairns, Blake and Dowd (2006) for an approach (CBD model) suited to old-age mortality (model coefficients of logistic model of $q_x$)
Lee-Carter Model (1)

- Mortality over time modeled using:
  \[ \log (u_{x,t}) = a_x + b_x k_t \]

- Mortality = average rate + rate of change \cdot time index

- Latter terms = variables that must be estimated from data and then multiplied

- Could use interaction term between the variables Year and Age but this specification would require \( t \times x \) effects to be fit compared to the \( t+x \) effects in the Lee-Carter model.

- => use non-linear/PCA regression (Brouhns, Denuit and Vermunt 2002; Currie 2016; Lee and Carter 1992)
Lee-Carter Model (2)

- Time index $k_t$ estimated for years within sample...
- ... so need to extrapolate $k_t$ for out-of-sample forecasts
- Time series models of varying complexity used to forecast $k_t$
- Density forecasts generated using realizations of forecast time series $k_t$
- Some studies also consider uncertainty of:
  - Time series model parameters
  - LC Model parameters
- **Two-step process** – fit model and extrapolate - common to other mortality models, such as CBD model
Extending the LC Model

- Single population
  - Cohort effect (Renshaw and Haberman 2006)
  - Smoothing time series (Currie 2013)

- What about multiple populations?

- Intuition = multi-population mortality forecasting model should produce more robust forecasts
  - Common factors (similar socioeconomic circumstances, shared improvements in public health and medical technology)
  - Common trends likely captured with more statistical credibility
  - => Li and Lee (2005) recommend even if interest is in single series
Two basic models

- Augmented Common Factor (Li and Lee 2005)
  \[
  \log (u_{x,t}) = a^i_x + b_x k_t + b^i x k^i_t
  \]

- Common Age Effect (Kleinow 2015)
  \[
  \log (u_{x,t}) = a^i_x + b_x k^i_t
  \]

- Not intended for large scale mortality forecasting - generally applied on smaller sub-set of data => judgment of modeler needed

- Hard to fit (complex optimization schemes/less known statistical techniques)

- Which specification is better, when, and why?
Taxonomy of multi-population models

- Diagram excerpted from Villegas, Haberman, Kaishev et al. (2017)
Another way?

• Explosion of interest in machine learning techniques
• For application within actuarial science (NL pricing), see Wüthrich and Buser (2018)
• Major success achieved on predictive modelling by Deep Learning in diverse fields, see LeCun, Bengio and Hinton (2015)
• Within actuarial science, review given by Richman (2018)
  • Talk tomorrow at 9:30 in ASTIN section
• Can we apply these techniques to the problem of large scale mortality forecasting?
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What is Machine Learning?

- To explain or predict? Shmueli (2010)
- Differences between statistical modelling (i.e. inference), and machine learning, due to distinction between tasks of predicting and explaining. Focus on predictive performance leads to:
  - Building algorithms to predict responses instead of specifying a stochastic data generating model (Breiman 2001)...
  - ... favouring models with good predictive performance that are often more difficult to interpret than statistical models.
  - Accepting bias in models if this is expected to reduce the overall prediction error.
  - Quantifying predictive error (i.e. out-of-sample error) by splitting data into training, validation and testing sets, or using by cross-validation.
What is Deep Learning?

• Traditional approach to modelling relies on manual model specification
  • time consuming/tedious
  • relies on expert knowledge
  • becomes difficult with very high dimensional data

• Representation Learning = allows algorithms automatically to design the model by specifying new covariates (Bengio, Courville and Vincent 2013)

• Deep Learning = representation learning technique that constructs complex models using deep hierarchies of learned covariates

• Deep Learning relies on neural networks; see Goodfellow, Bengio and Courville (2016)
Practical Successes of Deep Learning

• Computer vision starting with AlexNet architecture of Krizhevsky, Sutskever and Hinton (2012)

• Speech recognition (Hannun, Case, Casper et al. 2014).

• Natural language processing, e.g. Google’s neural translation machine (Wu, Schuster, Chen et al. 2016)

• Winning method in 2018 M4 time series forecasting competition (Makridakis, Spiliotis and Assimakopoulos 2018a).

• Analysis of GPS data (Brébisson, Simon, Auvolat et al. 2015)

• Analysis of tabular data (Guo and Berkhahn 2016) (plus other Kaggle competitions)
Single Layer NN = Linear Regression

- Single layer neural network
- Circles = variables
- Lines = connections between inputs and outputs
- Input layer holds the variables that are input to the network...
- ... multiplied by weights (coefficients) to get to result
- Single layer neural network is a linear regression!
Deep Feedforward Net

- Deep = multiple layers
- Feedforward = data travels from left to right
- Fully connected network = all neurons in layer connected to all neurons in previous layer
- More complicated representations of input data learned in hidden layers
- Subsequent layers represent regressions on the variables in hidden layers
Embedding layers

• Several specialized types of neural networks depending on purpose
• Key principle - Use architecture that expresses useful priors about the data => major performance gains
• Embedding layer learns dense vector transformation of sparse input vectors and clusters similar categories together; see Section 3.3 in Richman (2018)
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Extending LC – two perspectives

• Lee Carter model = regression model using features derived from data using PCA
  • CAE + ACF = regression models with features derived at a regional level

• Perspective 1: Use a neural network to model the regression problem and let it decide on the feature set

• Lee Carter model has a neural network formulation; see Richman and Wüthrich (2018)

• Perspective 2: use a more general step function formulation to specify the multi-population model

\[
\log(u_{x,t}) = g(x) + h(x)i(t),
\]

\[
g(x) = \begin{cases} 
    a_1 & \text{for } x = 1, \\
    a_2 & \text{for } x = 2, \\
    \vdots & \\
    a_\omega & \text{for } x = \omega,
\end{cases}
\]
Deep neural network

- Categorical inputs to network defined using embedding layers = vector valued step functions of parameters calibrated from input data
- Year input is numerical
- Intermediate layers combine the inputs into new features (128 nodes per layer) using non-linear transformations
- Deep networks hard to optimize => add a skip connection (He, Zhang, Ren et al. 2016)
Data from HMD

• Mortality data sourced from Human Mortality Database (HMD)
• Covers mortality rates for ~41 countries, for both genders, from 1950-2016
• Divide data into training and test sets:
  • Training set = observations at ages 0-99 occurring in the years before 2000
  • Test set = observations in the years 2000-2016
• Countries in the HMD that have at least ten years of data before year 2000 (excludes Germany, Croatia and Chile)
• 38 of the 41 countries used = aim to forecast 76 distinct sets of mortality rates
Female Mortality, USA, 1950-2016
Testing the models

• Test criterion = smallest MSE on out-of-sample mortality forecasts
  • MSE is natural choice
  • Optimization form of PCA/SVD uses MSE (Efron and Hastie 2016)
  • Maximises likelihood of Gaussian model

• Chose best model of each class for the tests:
  • Lee Carter fit with SVD
  • Lee Carter calibrated to regional mortality
  • Augmented Common Factor model
  • Common Age Effects model
  • Best of Deep Neural Networks

• Best deep network determined on forecasts in years 1990-1999
Choosing the best NN – 1990-1999
Performance – 2000-2016
Results

- Results of comparing the models
- Best performing model is deep neural network...
- ...produces the best out-of-time forecasts 51 out of 76 times
- for purposes of large scale mortality forecasting, deep neural networks dramatically outperform traditional single and multi-population forecasting models

<table>
<thead>
<tr>
<th>Model</th>
<th>Average MSE</th>
<th>Median MSE</th>
<th>Best Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 LC_SVD</td>
<td>5.50</td>
<td>2.48</td>
<td>7</td>
</tr>
<tr>
<td>2 LC_ACF_region</td>
<td>3.46</td>
<td>2.50</td>
<td>10</td>
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<tr>
<td>3 ACF_BP</td>
<td>6.12</td>
<td>3.00</td>
<td>4</td>
</tr>
<tr>
<td>4 CAE_BP</td>
<td>5.59</td>
<td>3.46</td>
<td>4</td>
</tr>
<tr>
<td>5 DEEP</td>
<td>2.68</td>
<td>1.38</td>
<td>51</td>
</tr>
</tbody>
</table>
Implicit Cohort Effects
Age Embedding
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Discussion

- Deep neural nets have enormous potential to solve model specification problems ... once a suitable deep architecture has been found

- Skip connections make a big difference since model only needs to learn residuals (He, Zhang, Ren et al. 2016)
  - See recent work by Gabrielli, Richman and Wüthrich (2018) and Schelldorfer and Wüthrich (2019)

- Against conventional wisdom: tanh was better than ReLU on this tabular data set

- Embeddings are a powerful way to understand and extend traditional statistical models
Conclusion

• One important comment we received stated that although neural network methods are a black box, their superiority in out-of-sample forecasting is clearly demonstrated.

• How can we give key stakeholders (including regulators) comfort around deep neural networks?
  • Interpretability - LIME (Ribeiro, Singh and Guestrin 2016)
  • Design the model for interpretability - Combined Actuarial Neural Net (CANN)– (Wüthrich and Merz 2018)

• Future research to consider:
  • Ensembling of models (56/76 by ensembling ReLU+ tanh)
  • Uncertainty bounds
References


Technical Details

- 5 dimensional embeddings
- Regularization - Dropout = 5%; see Srivastava, Hinton, Krizhevsky et al. (2014)
- Design - Batchnorm; see Ioffe and Szegedy (2015)
- Tried several combinations:
  - Non-linear function - ReLu vs tanh
  - Depth - 2 layers vs 5 layers
  - Design - no skip connection vs skip connection
Get involved

• Insurance Data Science conference - 14 June 2019
• ETH Zurich
• https://insurancedatascience.org/
• Amazing line-up of papers, presentations and speakers!

• Kasa.ai – launching soon, led by Kevin Kuo of Rstudio
• An open research group encouraging innovation in insurance analytics
• Some interesting projects planned
Thanks for listening - Any questions?

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